JOURNAL OF MATERIALS SCIENCE 19 (1984) 3407–3415

# Experimental observations of the strain distribution around a matrix crack bridged by reinforcing members and its effect on tensile fracture

J. G. MORLEY, I. R. McCOLL
*Wolfson Institute of Interfacial Technology, University of Nottingham, Nottingham, UK*

Measurements have been made of the elastic strain distributions developed around matrix cracks of various lengths bridged orthogonally by reinforcing members. The experimental specimens consisted of aluminium sheets reinforced with stainless steel tubes. The strains were produced by tensile loads applied parallel to the alignment of the reinforcing members. Measurements were made on reinforced and unreinforced specimens. For both conditions good agreement was found between the experimental data and the strain field previously postulated as a means to calculate critical strains for unstable crack extension. A comparison is made between the failure processes predicted by the analytical model and those observed experimentally in fibre composites.

## 1. Introduction

Most studies of tensile failure of unidirectionally reinforced fibrous composites have envisaged that, at relatively small composite strain values, some fibres fail at flaws or weak points. The flaws are considered to be distributed both in severity and position along the fibres. Thus the initial fibre failures occur randomly throughout the composite and their numbers increase as the load applied to the composite is increased. The load bearing ability of a fibre is lost at the point of fracture but this loss of strength is localized because stress is transferred from the matrix to the broken fibre over the fibre/matrix interface near the point of fracture. Thus at some little distance from the point of fracture the load bearing ability of the fibre within the composite is unimpaired. Estimates can be made of the length of the stress transfer distance. This approach to composite fracture was first considered by Rosen [1] who assumed that the load carried by a fibre before it fractured would be distributed uniformly amongst the intact fibres within the same cross section of the composite. Zweben and Rosen [2] examined the effect of local stress transfer to the nearest neighbours of a fractured fibre and considered the probability that this enhanced stress would cause one or more to fail. The stress enhancement has a maximum value in the plane of the initial fibre break and fails with increasing distance from this plane. The effect thus depends on the flaw distribution along the adjacent intact fibres. The cumulative failure theory has been further developed by Phoenix and co-worker [3–6]. Batdorf and co-worker [7, 8] and Barry [9] have developed alternative approaches. The cumulative fibre failure theories predict that composite failure will occur when regions are present within which several adjacent fibres have fractured.

In order to develop more comprehensive theories of the tensile fracture of fibre composites it is necessary to take various phenomena into account. Some observations of interest are as follows. Fuwa *et al.* [10] found that the strength of carbon fibre resin composites was approximately that of the average fibre strengths and was relatively independent of the strength of the matrix. This implies that the strength is governed largely by the properties of the fibres. However, the nature of the fracture surfaces was observed to

 © *1984 Chapman and Hall Ltd.* 

depend on the properties of the matrix, being brittle and localized in the case of a strong matrix and fibrous when the matrix was weak. When the matrix was strong and the failure localized, large numbers of adjacent fibre fractures were observed after composite fracture in regions remote from the fracture site.

When unidirectional glass fibre polymer matrix composites are subjected to a tensile load in an acidic environment, the fibres fracture in a very planar fashion. Cracks extending over 150 fibre diameters have been observed in these circumstances [11] and specimens containing these flaws were still able to support an appreciable stress. Also the strength of unidirectional composites is known to be insensitive to the presence of notches and cut outs which extend over many fibre diameters.

These phenomena can be accommodated by a crack propagation model which assumes that a matrix crack will propagate from a damaged region and will then become stabilized as it increases in length and becomes bridged by intact fibres [12]. This analysis is based on an assumed form of the strain field extending over appreciable distances on each side of a matrix crack which is considered to be bridged by reinforcing fibres [13]. Following Griffith [14] unstable crack growth is considered to occur when the rate of release of strain energy with increasing crack length exceeds the rate of energy absorption. Griffith was able to calculate the strain energy released by an isotropic elastic material containing an elliptical crack by integrating the strain field surrounding the crack derived from elasticity theory. This approach is not available in the case of cracks bridged by fibres because of the increased complexity of the situation and it is necessary to make some simplifying assumptions in order to deduce the approximate form of the strain field. Having done so the rate of release and absorption of energy with increasing crack length can be calculated.

In this paper experimental data on the form of the strain field surrounding a crack in an elastic sheet bridged by reinforcing members is presented. Good agreement between the analytical model and the experimental data is observed for this particular system. The relevance of the strain field data presented here to the tensile failure of unidirectionally reinforced composites is discussed.

## 2. The analytical model

The analytical model has been described in detail elsewhere [13] and will be given in outline only here. A unidirectional system loaded in tension in the direction of fibre alignment is assumed. A matrix crack perpendicular to the fibres and bridged by the fibres is assumed to be present. The basic assumption is that an elliptical zone around the crack exists within which the strain field is perturbed by the crack. The size of the elliptical zone is independent of the volume fraction of the fibres but the fibres, when present, modify the strain distribution within the elliptical zone. The crack forms the minor axis of the elliptical zone whose major axis is three times the crack length. When reinforcing fibres are not present the strain carried by the material is assumed to be zero at the crack face and to increase linearly in the direction of the applied load. At the edge of the elliptical zone the strain carried by the material reaches the value of the bulk strain $\epsilon_\beta$ carried by the material in regions remote from the crack (Fig. 1). The strain gradient thus has a minimum value at the centre of the crack and it increases as the crack tip is approached becoming infinite at the crack tip. The rate of release of strain energy with increasing crack length is computed from the model by dividing the crack into a number of parallel segments each assumed to be independent of each other and to be subjected only to a tensile load applied in the direction of loading. The strain energy contained in a strip is then readily calculated and the strain energy contained within the elliptical zone is obtained by summing the strain energy contained in the individual strips. The calculation is then repeated after an assumed small increase in crack length and the rate of release of strain energy with increasing crack length obtained by numerical differentiation.

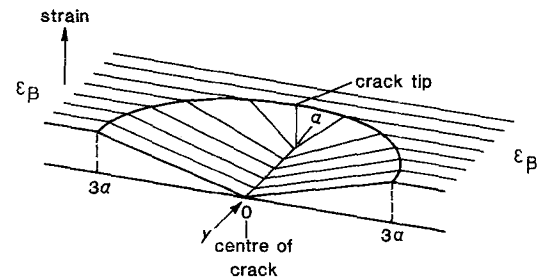



*Figure 1* Assumed strain field developed around one half of a crack. Elastic strains are measured on the vertical axis.

This procedure gives the same critical strain $\epsilon_{\beta c}$ for unstable crack extensions as that predicted

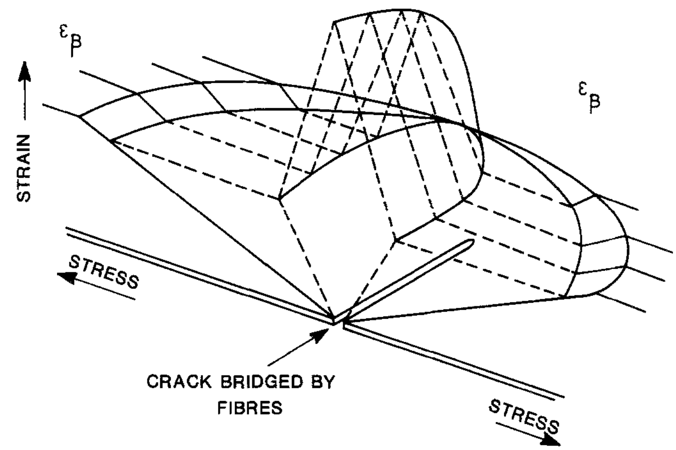



*Figure 2* Schematic illustration of one half of the assumed strain field developed around a matrix crack bridged by reinforcing fibres. Strain values plotted on the vertical axis. Full lines indicate the strain field which would be developed in the absence of the reinforcing fibres.

by the Griffith equation:

$$\epsilon_c = (G/E\pi a)^{1/2} \tag{1}$$

where $G$ is the work of fracture of the material, $E$ is its Young's modulus and $a$ is the half crack length. Here $\epsilon_c$ corresponds to $\epsilon_{\beta c}$.

When crack bridging fibres are present they carry an enhanced load at the crack face. This excess load is transferred back to the matrix so that the strain carried by the fibres decreases with increasing distance from the crack face (Fig. 2). The strain gradient in the fibres is given by:

$$\frac{d\epsilon_f}{dx} = \frac{-2\tau}{E_f r} = Q \tag{2}$$

where $\tau$ is the shear stress transfer occurring at the fibre matrix interface, $2r$ is the fibre diameter and $E_f$ is the fibre elastic modulus.

The corresponding strain gradient, $P$, developed in the matrix due to this transfer of load from the fibres is given by:

$$P = \frac{2V_f\tau}{E_m(1-V_f)r} \tag{3}$$

where $V_f$ is the fibre volume fraction and $E_m$ is Young's modulus of the matrix. This has to be added to the strain gradient which would have been carried by the unreinforced matrix so that, in the region of the crack the strain gradient carried by the matrix is,

$$\frac{d\epsilon_m}{dx} = \frac{2V_f\tau}{E_m(1-V_f)r} + \frac{\epsilon_\beta}{L_3} \tag{4}$$

where $L_3$ is the distance measured from the crack face to the edge of the elliptical zone for the particular strip considered.

At some point the strain carried by the fibres and the matrix in the strip will be the same. This is defined as the point $L_1$ in Fig. 3 and the strain at the point is $\epsilon_r$. It is assumed that no further stress

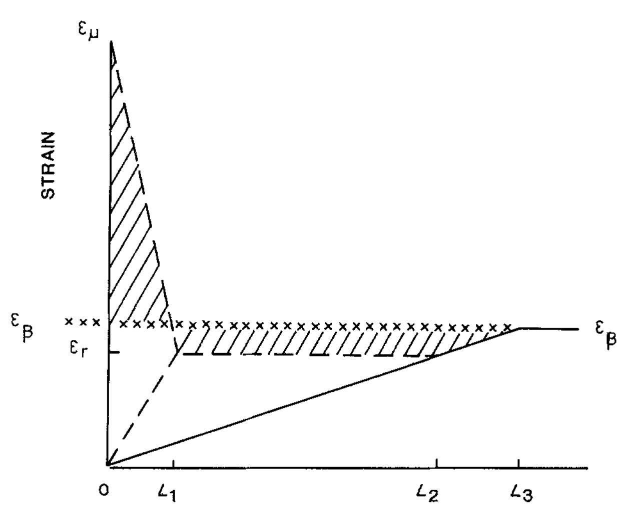



*Figure 3* Section through a quadrant of the strain field. Uniform strain in composite with uncracked matrix indicated × × × ×. Shaded areas indicate the amount of fibre extension and contraction generated by the matrix crack.

transfer takes place between fibres and matrix at distances greater than $L_1$ from the crack face. The strain carried by both fibres and matrix is assumed to remain constant until a point $L_2$ is reached on the strain gradient corresponding to the unreinforced strip. The strain carried by both fibres and matrix is then assumed to follow this strain gradient to the edge of the elliptical zone at the position $L_3$ (Fig. 3).

Since all strain perturbations are assumed to be confined within the boundaries of the elliptical zone it follows that the length of a reinforcing fibre which traverses the zone should be the same whether or not a matrix crack is present. In the vicinity of the crack the fibre carries an enhanced strain. Further away from the crack it has relaxed to some extent. The two areas shown shaded in Fig. 3 are proportional to the corresponding extension and contraction of the fibre and must therefore be equal. Sufficient data is now available to solve analytically the geometry of Fig. 3, and the relationships between the various quantities are as follows:

$$\begin{aligned} \epsilon_{\mathrm{r}} &= \{\epsilon_\beta L_3/[Q(P+\epsilon_\beta/L_3)^{-2}+L_3/\epsilon_\beta]\}^{1/2} \\ \epsilon_\mu &= L_1(P+Q+\epsilon_\beta/L_3) \\ L_1 &= \epsilon_{\mathrm{r}}(P+\epsilon_\beta/L_3)^{-1} \qquad (5) \\ L_2 &= L_3\epsilon_{\mathrm{r}}/\epsilon_\beta \\ L_3 &= 3(a^2-y^2)^{1/2} \end{aligned}$$

where $y$ is the distance measured from the centre of the crack to the centre of the particular strip.

From these relationships the strain energy carried by fibres and matrix in any given strip within the elliptical zone can be calculated. Thus the strain energy contained within the entire zone and the rate of release of strain energy with increasing crack length can be computed.

Energy is absorbed by differential movement between fibres and matrix over the distance $0L_1$ (Fig. 3). In the case of fibre reinforced polymers this will occur by frictional losses. Also energy can be absorbed by chemically debonding the fibre matrix interface over the distance $0L_1$, prior to the frictional losses occurring. The effects of these processes can be computed and taken into account in calculating the critical strain for matrix crack growth. Under some circumstances the crack could become unstable because the peak stress $\epsilon_\mu$ carried by the fibres (Fig. 3) causes a sufficient number of them to fail. This situation has been examined recently [12].

The validity of these calculations depends on the degree of correspondence between the analytical model and the actual form of the strain field around a crack bridged by reinforcing fibres. In this paper experimental data showing the form of the strain field for one type of fibre reinforced composite are given. This was constructed from a sheet of aluminium alloy containing a central crack bridged orthogonally by steel reinforcing members. Strain gauges were fixed to the aluminium sheet and the strain distributions developed by the application of a tensile load were measured. The strain distributions around cracks in unreinforced sheets were also measured. Good correlation between the experimental data and the analytical model is obtained for this particular experimental system.

## 3. Experimental details

### 3.1. Specimen fabrication

The details of the methods used to construct the experimental samples have been given elsewhere [15]. The material used was a clad aluminium alloy to BS 2L72, which had a nominal thickness of 0.265 mm. It was cut into panels 620 mm by 152 mm with the larger dimension parallel with the rolling direction. A slot 12 mm long was cut in the centre of the specimen aligned perpendicularly to the longer dimension of the sheet. Twenty stainless steel thin walled hyperdermic tubes having an outside diameter of 1.06 mm were bonded to both sides of the aluminium sheet parallel to the longer dimension using an epoxy adhesive. They were equally spaced 15 mm apart with ten tubes on each side of the sheet. The volume fraction of the tubes, measured on the basis of their outside diameter was 0.292. Their effective elastic modulus (tube i.d. 0.795 mm, $E =$ 170 GPa) was 75 GPa as was the Young's modulus of the aluminium sheet. Five rows of strain gauges were bonded to the sheet spaced at 20 mm intervals halfway between the reinforcing tubes. The strain gauges were arranged so that the strains could be measured within one quadrant of the strain field surrounding the crack in the sheet. The distribution of the strain gauges is shown in Fig. 4. Gauge line 1 corresponded to the major axis of the elliptical zone around the crack. The same strain gauge array was used for the unreinforced aluminium alloy sheets.

The output from the strain gauges was fed to a multi-channel data logger which provided a digital readout of the output of all of the strain gauges when a tensile load was applied to the ends of the specimen in the direction of the reinforcing members.

### 3.2. Experimental method

The strain gauges were first adjusted to give zero output with the specimen mounted between beam grips in a Servo Hydraulic Universal Testing Machine with zero applied load. A zero tension cyclic load was then applied to the specimens. This caused a fatigue crack to propagate from each end of the slot initially cut in the aluminium sheet. At various times during the growth of the fatigue crack the cyclic loading was interrupted and the specimen subjected to a static tensile load which was applied in a number of steps and removed in a similar manner. After each step increase or decrease of load the strain distribution was recorded. During the progress of the experiments some strain gauges indicated small compressive or tensile residual strains when the external load was reduced to zero indicating some permanent distortion.

## 4. Experimental results

In Figs. 5a, 6a and 7a the reversible elastic strains produced by an external load and measured at various positions of an unreinforced aluminium sheet are shown. The figures correspond to three central cracks, sizes 20, 40 and 71 mm and one quadrant of the strain field is shown. The form of the strain field derived from the theoretical model is also shown on these figures. It is apparent that the model agrees approximately with the experimentally observed strain field for all of the crack lengths examined. The differences between the model and the experimental observations can be summarized as follows. Firstly, along a line corresponding to the major axis of the elliptical zone around the crack, a greater elastic relaxation is observed than is predicted by the model. On the other hand near the crack tips the amount of elastic relaxation observed is less than that predicted by the model. In the construction of the model, the assumption is made that there is no stress enhancement beyond the crack tip. Experimentally, as would be expected, stress enhancement is observed in this region.

In Figs. 5b, 6b and 7b the measured strain fields around cracks of size 20, 40 and 71 mm, respec-

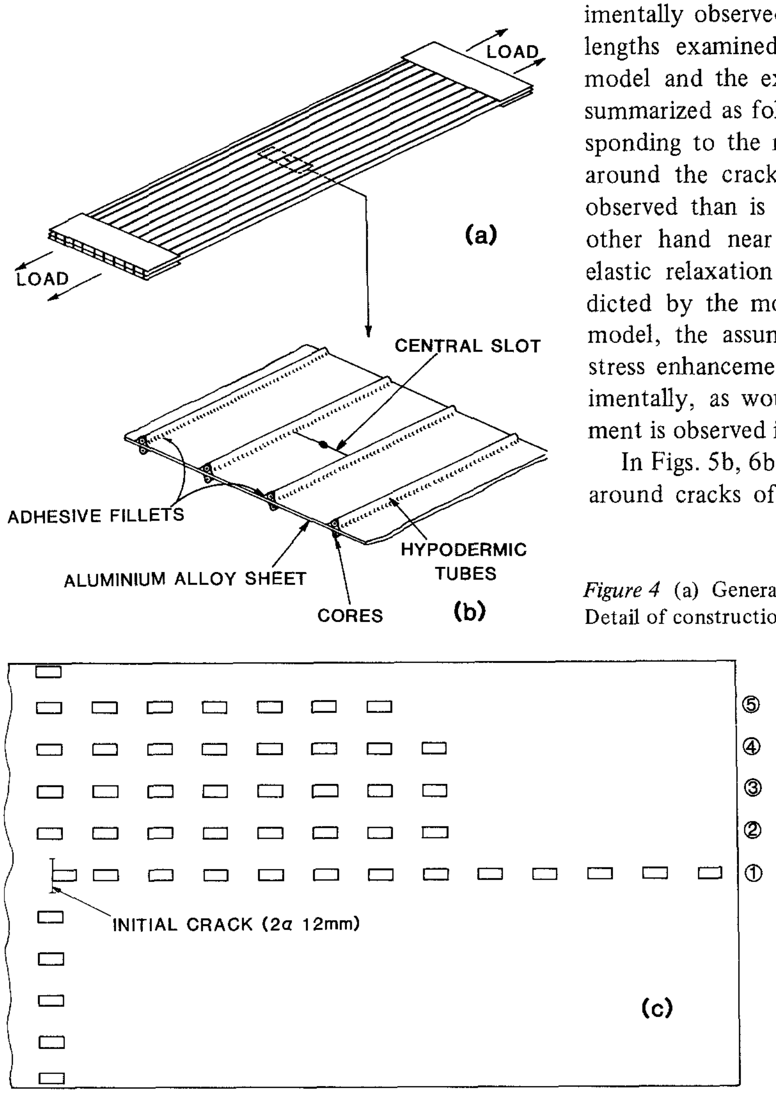



*Figure 4* (a) General arrangement of test specimen. (b) Detail of construction. (c) Distribution of strain gauges.

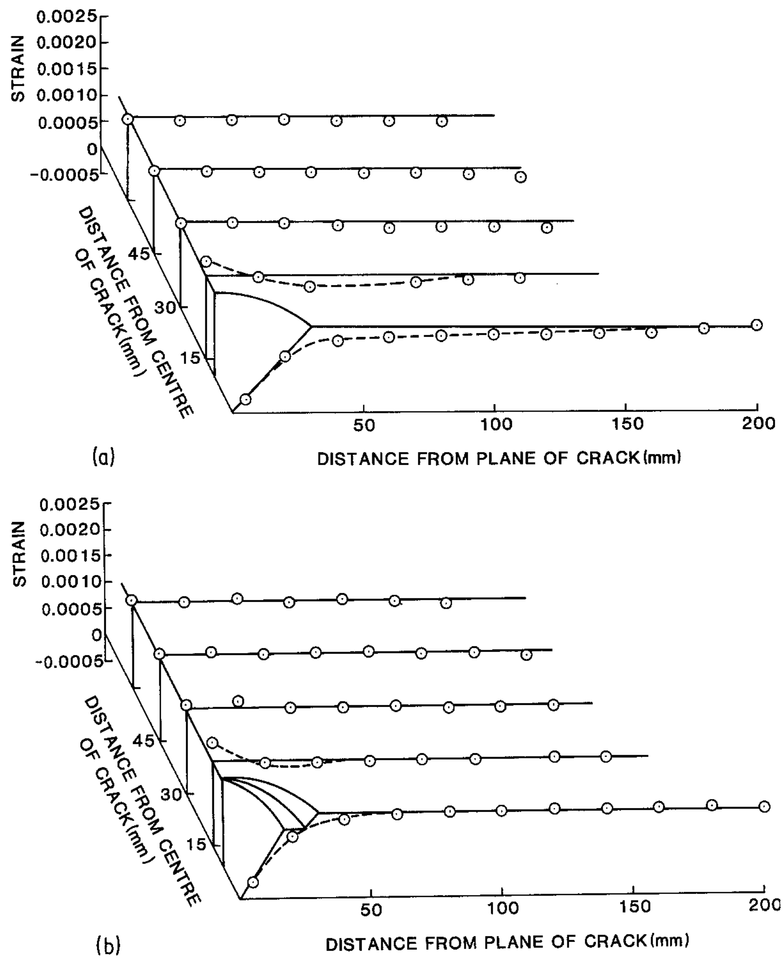



*Figure 5* (a) Strain distributions around unreinforced crack. Crack length = 20 mm $\epsilon_\beta = 1600\,\mu\epsilon$. (b) Strain distribution around reinforced crack. Crack length = 20 mm $\epsilon_\beta = 1600\,\mu\epsilon$. ⊙ = observed strains.

tively, bridged by reinforced members are given. As predicted by the model the amount by which the matrix sheet relaxes is reduced, particularly in regions near the centre of the crack. It is apparent, from the data given, that the effectiveness of the reinforcing members in inhibiting the relaxation of the matrix increases as the length of the crack increases. This is in agreement with the predictions of the analytical model. When the crack length is small $\epsilon_\beta/L_3$ becomes the dominant term on the right hand side of Equation 4 indicating that the reinforcing members have little influence on the form of the strain field under these conditions.

The stress transfer occurring between the reinforcing fibres and the matrix was computed from the change in strain in the aluminium sheet brought about by the presence of the reinforcing members. The change in strain in the vicinity of the crack along gauge line number 1 was calculated for crack lengths of 40 and 71 mm from Figs. 6a and b and Figs. 7a and b. The computed change in strain gradient was then taken as being equal to $P$ in Equation 3 thus enabling the effective value of $\tau$, the rate of shear stress transfer between the crack bridging reinforcing members and the aluminium sheet to be estimated. The value of $\tau$ calculated in this way proved to be about 1 MPa and the calculation assumes that the stress is being transferred over the entire surface of the reinforcing tubes. This computed value is similar to the accepted values for conventional fibre reinforced polymers ($\sim$ 5 MPa) when it is remembered that only about one quarter of the surface of the steel tube was effectively bonded to the surface of the aluminium sheet.

An effective shear stress transfer value $\tau$ of 1 MPa has been used to calculate the strain profile along gauge lines 1, 2, 3, 4 and 5 for each of the three reinforced sheets (Figs. 5b, 6b and 7b). The observed strains developed in the aluminium sheets agree very well with the predictions of the model. Discrepancies between the theoretical strain values

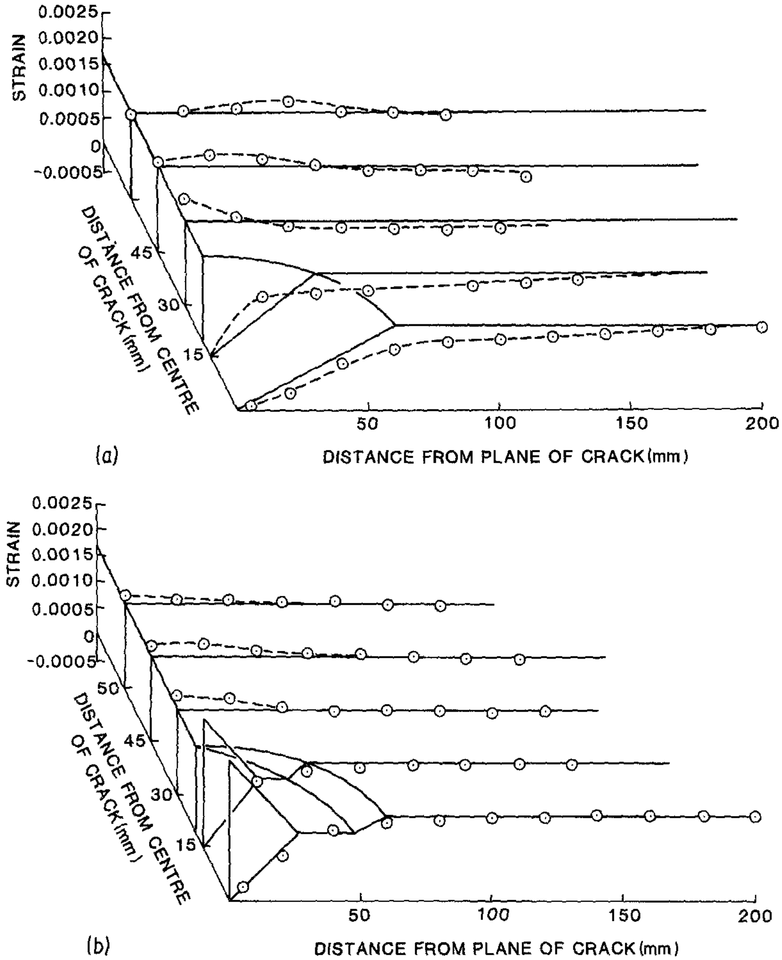



*Figure 6* (a) Strain distributions around unreinforced crack. Crack length = 40 mm $\epsilon_\beta = 1600\,\mu\epsilon$. (b) Strain distribution around reinforced crack. Crack length = 40 mm $\epsilon_\beta = 1600\,\mu\epsilon$. ⊙ = observed strains.

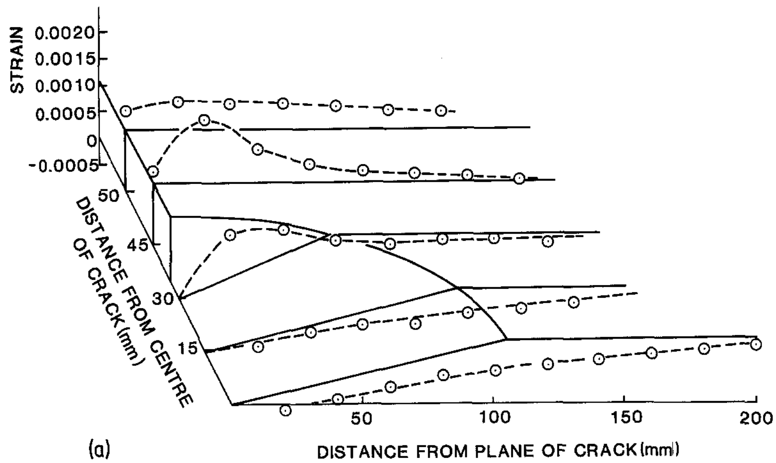
again occur in regions near the crack tip but these are much reduced compared with those observed for the case of the unreinforced crack.

The strain carried by the crack bridging reinforcing members in the vicinity of the crack, computed from the analytical model, are also shown in Figs. 6b and 7b. Since the strains carried by the reinforcing members were not measured directly these values were not confirmed by direct experiment.

## 5. Discussion

It is apparent that the analytical model of the strain field surrounding a matrix crack bridged by reinforcing members previously proposed agrees quite well with the data obtained from the particular experimental system studied here. Indirect agreement between the analytical model and this type of experimental system has previously been obtained through the close agreement between the calculated and observed strain values for unstable crack growth in sheet metal specimens reinforced with crack bridging reinforcing members [13]. The simple two-dimensional analysis presented here therefore appears able to predict with fair accuracy the strain at which a crack in an isotropic elastic material will propagate when bridged by reinforcing members.

Recently the model has been extended to analyse the stability of cracks bridged by fibres which have a distribution of strengths [12]. This analysis indicates that the flaws originating as a number of adjacent fibre breaks can propagate as a matrix crack at low stress levels. However, as the matrix crack propagates, it encounters and by passes intact fibres. These bridge the crack and inhibit its growth so that the crack eventually stops. Stable growth then takes place as the stress carried by the composite is increased. Crack growth under these conditions can be analysed quite straightforwardly using the model described here if it is assumed that the broken fibres can be



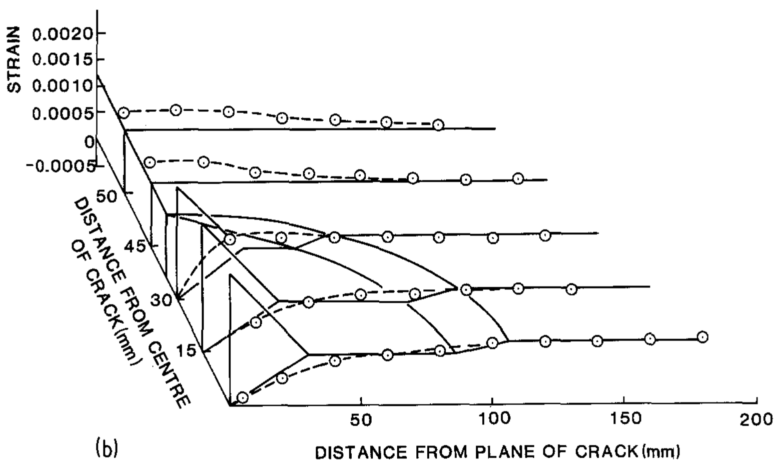




*Figure 7* (a) Strain distributions around unreinforced crack. Crack length = 71 mm $\epsilon_\beta = 1200\,\mu\epsilon$. (b) Strain distribution around reinforced crack. Crack length = 71 mm $\epsilon_\beta = 1200\,\mu\epsilon$. ⊙ = observed strains.

regarded as being distributed uniformly along the length of the crack. The broken fibres then form part of the "matrix" and enhance its elastic modulus. As the load carried by the composite increases, a strain value is reached at which some of the fibres bridging the crack fail at their fracture strains. This proportion will depend on the average strain carried by the crack bridging fibres and on the distribution of fibre strengths. It will increase as the composite strain increases. Eventually at some critical strain value unstable crack growth will occur by the sequential failure of the still intact crack bridging fibres. At this point the assumption of a uniform distribution of fibres along the length of the crack is justified since the size of the initial flaw is small compared with the length of the crack which eventually becomes unstable. This analysis has been shown to predict the failing strains of unidirectional carbon fibre composites quite accurately [12].

In these computations no account is taken of the anisotropic nature of the "matrix" which is formed from the polymeric binder plus the fibres which have fractured at the crack face prior to unstable crack growth. An improved agreement with experiment would therefore be expected by modifying the form of the strain field used in the model to enable it to correspond more closely with these circumstances. The modification would involve an increase in the major axis of the elliptical zone assumed to be present around the crack (see Denby and Leendertz [16]). An increase in the major axis of the elliptical zone would also require a corresponding change in the strain gradient and the shape of the partially relaxed zone around the crack.

The mechanism of stable crack growth described above implies the occurrence within a composite of stable cracks of appreciable dimensions along the plane of which fibre fractures will be present. Many such cracks would be expected within a composite and some of them on slightly different planes might be expected to link via shear failure of the matrix. Failed regions of this type were observed by Fuwa *et al.* [10]. It was shown by Morley [12] that an analysis based on the strain field model predicts that the more strongly carbon fibres are bonded to the matrix the more sensitive the strength of the composite to the size of an initial flaw. Thus more strongly bonded composites would be expected to fail by the unstable growth of a single large crack, or small number of large cracks, and so exhibit the brittle mode of failure which is observed. Conversely cracks in weakly bonded composites are bridged by a large proportion of the fibres at instability and are less sensitive to the size of the initial flaw. Thus failure might be expected to occur from a multiplicity of small cracks and show an irregular fracture face. In the limit, as the coupling between the fibres and matrix becomes negligible the composite strength would be expected to approach the bundle strengths of the fibres. The model predicts that very localized enhanced stresses are carried by the crack bridging fibres in the plane of the crack. This would be expected to promote their attack in regions close to the crack plane in an aggressive chemical environment. Thus, the planar fractures faces observed under stress corrosive conditions [11] are consistent with the analysis.

One fundamental assumption in the analysis presented here is that the strain energy release rate during crack extension can be computed simply from the strains developed in the direction of fibre alignment. There is some justification for this argument in that the fibres make the major contribution to the elastic modulus of the composite in this direction so that the strain energy associated with deformation in other directions will be small by comparison. This simplifying assumption seems acceptable on the grounds that the model in its present form predicts the numerical values of fracture strains quite accurately and also encompasses a number of factors which influence the nature of the fracture process.

## 6. Conclusions

A simple physical model of a crack in an isotropic elastic material bridged by reinforcing members has been constructed and the strain field developed around the crack has been measured directly. The experimental values found agree quite well with the strain values predicted by a simple analytical model of the strain field and the rate of stress transfer between the reinforcing members and the matrix is in agreement with the expected values.

The critical strain values for the unstable growth of a matrix crack when bridged by reinforcing members calculated from the analytical model have previously been observed to agree closely with experimental values so that the model appears to describe transverse crack growth in a reinforced isotropic matrix quite well. The analytical model can also be used to predict failure processes with reasonable accuracy in composite systems in which the matrix is not elastically isotropic.


## Acknowledgements

The work reported here was carried out with the support of the Science and Engineering Research Council. We are indebted to Mr A. Martin for his assistance in carrying out the experimental investigation.


## References


1. B. W. ROSEN, *AIAA J.* **2** (11) (1964) 1985.
2. C. ZWEBEN and B. W. ROSEN, *J. Mech. Phys. Solids* **18** (1970) 189.
3. D. G. HARLOW and S. L. PHOENIX, *J. Comp. Mater.* **12** (1978) 195.
4. *Idem, ibid.* **12** (1978) 314.
5. *Idem, Int. J. Fract.* **17** (1981) 347.
6. *Idem, ibid.* **17** (1981) 601.
7. S. B. BATDORF, *J. Reinf. Plast. Compos.* **1** (1982) 153.
8. S. B. BATDORF and R. GHAFFARIAN, *J. Mater. Sci. Lett.* **1** (1982) 295.
9. P. W. BARRY, *J. Mater. Sci.* **13** (1978) 2177.
10. M. FUWA, A. R. BUNSELL and B. HARRIS, *ibid.* **10** (1975) 2062.
11. B. NOBLE, S. J. HARRIS and M. J. OWEN, *ibid.* **18** (1983) 1244.
12. J. G. MORLEY, *ibid.* **18** (1983) 1564.
13. I. R. McCOLL and J. G. MORLEY, *Phil. Trans. Roy. Soc.* **287** (1338) (1977) 17.
14. A. A. GRIFFITH, *ibid.* **A221** (1920) 163.
15. I. R. McCOLL and J. G. MORLEY, *Int. J. Fract.* **18** (1982) 191.
16. D. DENBY and J. A. LEENDERTZ, *J. Strain Anal.* **9** (1974) 17.



*Received 18 December 1983*
*and accepted 16 January 1984*